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Investigation of various orthogonal wavelets for precise analysis of X-ray images

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ABSTRACT
Now-a-days X-rays are playing very important role in medicine. One of the most important applications of X-ray is detecting fractures in bones. X-ray provides important information about the type and location of the fracture. Sometimes it is not possible to detect the fractures in X-rays with naked eye. So it needs further processing to detect the fractures even at minute levels. To detect minute fractures, in this paper various edge feature extraction methods are analyzed which helps medical practitioners to study the bone structure, detects the bone fracture, measurement of fracture treatment, and treatment planning prior to surgery. The classical derivative edge detection operators such as Roberts, Prewitt, sobel, Laplacian of Gaussian can be used as edge detectors, but a lot of false edge information will be extracted. Therefore a technique based on orthogonal wavelet transforms like Haar, daubechies, coiflet, symlets are applied to detect the edges and are compared. Among all the methods, Haar wavelet transform method performs well in detecting the edges with better quality. The various performance metrics like Ratio of Edge pixels to size of image (REPS), peak signal to noise ratio (PSNR) and computation time are compared for various wavelets.
Keywords - Edge detection, Multi-resolution analysis, orthogonal wavelet, Wavelet transforms

I. INTRODUCTION

Edge feature extraction of X-ray bone image is very useful for the medical practitioners as it provides important information for diagnosis which in turn enable them to give better treatment decisions to the patients. Presently digital images are increasingly used by medical practitioners for disease diagnosis The images are produced by several medical equipments like MRI, CT, ultrasound and X-ray. Out of these, X-ray is the oldest and frequently used devices, as they are painless and economical[2].The X-ray images are used during various stages of treatment which include fracture diagnosis and treatment.

Edge feature extraction deals with extracting or detecting the edge of an image. It is the most common approach for detecting meaningful discontinuities in the gray level [11]. It is one of basal contents in the image processing and analysis, and also is a kind of issues which are unable to be resolved completely so far [3]. When image is acquired, the factors such as projection, mix, and noise are produced. These factors bring on image feature's blur and distortion, consequently it is very difficult to extract image feature. Moreover, due to such factors it is also difficult to detect edge[9].

The classical derivative operators such as Roberts, Prewitt,sobel, Laplacian of Gaussian[4][6] can be used. But a lot of false edge information will be extracted. They are also sensitive to noise. These operators are applied on pixel by pixel basis. They are slow in operation. The high frequency components of an image includes both edges and noise. So detecting the edge is not an easy task. Therefore an efficient technique based on wavelet transform is used to detect the edges. This is because wavelet transform has the advantage of detecting edges using different scales. This paper mainly discusses about applications of various orthogonal wavelet transforms to X-ray bone image for edge detection. Haar wavelet transform is selected as an best edge detector due to its superior ability in detecting the edges and extraction multiscales.

II. WAVELET TRANSFORM THEORY

Frequency domain analysis using fourier transform is extremely useful for analyzing the signal because the signal's frequency content is very important for understanding the nature of signal and the noise that contaminates it. The only drawback is, loss of time information. When looking at a fourier transform of signal, it is impossible to tell when a particular event took place. So to overcome this drawback, the same transform was adapted to analyze only a small window of the signal at a time. This technique is known as short time fourier transform which maps a signal into a two dimensional function of time and frequency to get the localized point information but the only drawback is that, the window size is same for all frequencies. Many signals require more flexible approach that is flexible window size. This technique is known as wavelet transform. It allows the different window sizes for different frequencies. It allows the use of long time intervals for more-precise low frequency information and short time interval for high –frequency information. The major advantage of wavelets is the ability to perform localized area of a larger signal

A wavelet is a "small wave", which has its energy concentrated in time to give a tool for the analysis of transient, non-stationary, or time-varying phenomena about analyzing signal with short duration finite energy functions[1]. They transform the signal under investigation into another representation which presents the signal in a more useful form. It still has the oscillating wave-like characteristic but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation. Wavelet transform is widely used in image processing. Wavelet transform of a signal means to describe the signal with a family of functions.

The wavelet transform of continuous time signal is known as continuous wavelet transform (CWT) and is defined as [1][10]

$$W(a,b) = f(x) * \psi_{a,b}(x) = \int_R f(x) \tfrac{1}{\sqrt{a}} \psi(\frac{x-b}{a}) dx \quad (1)$$

Where $\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi(\frac{x-b}{a})$

'a' is the scaling parameter, and 'b' is the time shift parameter. The signal f(x) is transformed by analyzing function $\psi(\frac{x-b}{a})$

The analyzing function is not limited to complex exponential as used in the fourier transform. In fact, the only restriction is that, it must be short and oscillatory. This restriction ensures that the integral in the above equation is finite and hence the name wavelet or small wave was given to this transform. The continuous wavelet transform is a function of two parameters and therefore, contains a high amount of redundant information when analyzing the signal. Instead of continuously varying the parameters, the signal with a small number of scales with varying number of translations at each scale are analyzed. This is the discrete wavelet transform. The sampling of CWT is the discrete wavelet transform which is given by the following equation

$$w(j,k) = \int_t f(t) 2^{j/2} \Psi(2^j t - k) dt$$

$$here$$

$$a = 2^{-j}, b = 2^{-j} k \quad (2)$$

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$$here$$

$$a = 2^{-j}, b = 2^{-j} k \quad (2)$$

Here j, k are integers representing the set of discrete translations and discrete dilations. If the signal is an image, the one dimension wavelet transform should be extended to two-dimensional wavelet transform. For a two-dimensional function *f(x, y)*, the wavelet transform is

$$Wf(x,y) = f(x,y) * \psi_{a,b}(x,y) \quad (3)$$

Where,*expresses the convolution along different direction, *s* is the scale. In two-dimensional images, the intensity of edges can be enhanced in each one-dimensional image. If the window of the images is convolved in the *x* direction over an image, a peak will result at positions where an edge is aligned with the *y* direction. This operation is approximately like taking the first derivative of the image intensity function with respect to *x* or *y*.

III. WAVELET DECOMPOSITION

As a mathematics tool, the wavelet transform (WT) brings revolutionary influence on signal analysis, image processing, and other research fields of nonlinear science. Compared to traditional Fourier transform, wavelet transform provides image's partial signal characteristics in spatial and frequency domain, but the Fourier transform only reveals an image's frequency attributes. Thus, wavelet transform can position accurately discontinuous point (high frequency details) in edge detection, thereby extracting edges of images.In the discrete wavelet transform, an image signal can be analyzed by passing it through an analysis filter bank followed by a decimation operation. This analysis filter bank consists of a low pass and a high pass filter at each decomposition stage. When a signal passes though these filters, it splits into two bands. The low pass filter, which corresponds to averaging information, extracts the coarse information of the signal. The high pass filter, which corresponds to a differentiation operation, extracts the detail information of the signal. The output of the filtering operation is decimated by two

LL1	HL1
LH1	HH1

Fig.1.one level image decomposition using DWT

A two dimensional transform can be accomplished by performing two separate one-dimensional transforms. First, the image is filtered along x-dimension using low pass and high pass analysis filters and decimated by two. Low pass filtered coefficients are stored on the left part of the matrix and high pass filtered on the right. Because of decimation, the total size of transformed image is same as the original image. Then, it is followed by filtering the sub image along the y-direction and decimated by two.

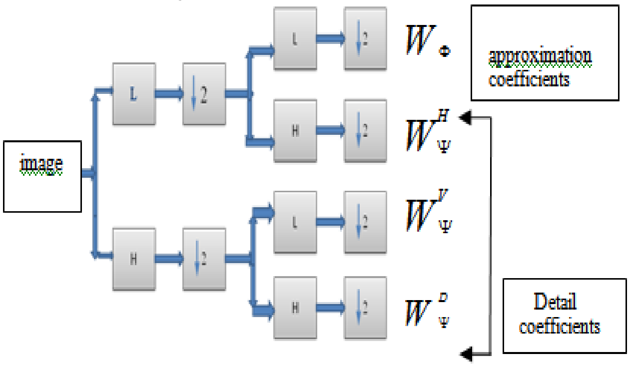


Fig.2. wavelet filter bank for one-level image decomposition

IV. ORTHOGONAL WAVELETS

The discrete wavelet transform have two functions i.e scaling function and wavelet function. The wavelet functions are derived from the scaling function. The scaling function is orthogonal to wavelet function and hence they are called orthogonal wavelets. The support interval of wavelet is the range of the interval over which the scaling and wavelet function is defined. These are finite support and compact wavelets which are more popular due to their relations to multiresolution filter banks.Orthogonal wavelet systems decompose signals into well-behaved orthogonal signal spaces. However, the analysis and synthesis filters are not symmetric[8].There are different types of orthogonal wavelets such as Haar, daubechies, coiflets, symlets etc.

(a)Haar Wavelet

The Haar wavelet is basic and simple wavelet[5].Haar scaling function defined as $\phi(t)$

$$\phi(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{else} \end{cases} \tag{4}$$

Haar wavelet function $\psi(t)$ is given by [5]

$$\psi(t) = \begin{cases} 1 & 0 \le t \le 1/2 \\ -1 & 1/2 \le t \le 1 \\ 0 & \text{else where} \end{cases} \tag{5}$$

$$\phi(t) = \phi(2t) + \phi(2t-1) \tag{6}$$

$$\psi(t) = \phi(2t) - \phi(2t-1)$$

$$\phi(t) = \sum_{k=0}^{1} h(k) * \sqrt{2}\phi(2t-k)$$

$$\psi(t) = \sum_{k=0}^{1} g(k) * \sqrt{2}\phi(2t-k)$$

Where $h(0) = 1/\sqrt{2}, h(1) = 1/\sqrt{2}, g(0) = 1/\sqrt{2}, g(1) = -1/\sqrt{2}$ (7)

h(k) gives relationship between j^{th} level scaling basis function to $(j-1)^{th}$ level scale basis function. g(k) gives relationship between j^{th} level wavelet basis function to $(j-1)^{th}$ level wavelet basis function. h(0),h(1) are low pass scaling filter coefficients , g(0),g(1) are high pass wavelet filter coefficients. Since it contains two elements in h(k),g(k) haar wavelet is simple wavelet

(b)Daubechies wavelet

These are compactly supported orthogonal wavelets thus making discrete wavelet analysis practicable. There are different types of daubechies wavelets[15].They are db2,db3,db4,db5,db6,db7, ,db8,db9,db10.Here the number indicates the number of filter coefficients. For example, in db3, the scaling function is expressed in terms of three filter coefficients

$$\phi(t) = h(0)\sqrt{2}\phi(2t) + h(1)\sqrt{2}\phi(2t-1) + h(2)\sqrt{2}\phi(2t-2) \tag{8}$$

where $\phi(t)$ is expressed in terms of $\phi(2t)$ and its translates.Daubechies wavelet function can be expressed in terms of g(k) and they can be represented by wavelet function $\psi(t)$ .The wavelet function are fractal in nature and iterative methods are required to see its shape.

using converged $\phi(t)$ values to obtain $\psi(t)$ using the relation

$$\psi(t) = g(0)\sqrt{2\phi}(2t) + g(1)\sqrt{2\phi}(2t-1) + g(2)\sqrt{2\phi}(2t-2) \tag{9}$$

(c) Coiflets:

Coiflet wavelets are obtained by imposing vanishing moment conditions[14] on both scaling and wavelet functions. More conditions on vanishing moments(p) imply more filter coefficients. There are 2p number of vanishing moment conditions imposed on wavelet function, 2p-1 on scaling function. There are different types of coiflets. They are coif1,coif2,coif3,coif4,coif5.Here the number indicates the order of the wavelet and it indicates the number of low pass filter and high pass filter coefficients. For example in coif2,there are two low pass filter and two high pass filter coefficients. The number of filter coefficients can be changed in order to get the best result

(d)Symlets:

The symlets are nearly symmetrical wavelets. These wavelets are orthogonal, compact support, nearly linear phase and it has p vanishing moments. There are various types of symlets. They are sym2,sym3,sym4,sym5,sym6,sym7,sym8. Here the number indicates the order of the wavelet and it indicates the number of low pass filter and high pass filter coefficients. For example in sym2,there are two low pass filter and two high pass filter coefficients. The order of the wavelet can be changed in order to get the best result.

V. EDGE DETECTION

Edge detection is very important in the digital image processing, because the edge is boundary of the target and the background. The target and the background can be differentiated from the edges. But the borderline detected may produce interruption as a result of existing noise and image dark.

(a) procedure for wavelet based edge detection

(1) Consider the image for which, the edge is to be detected.Apply the various types of orthogonal wavelet transforms like Haar, debauchies, symlets coiflets to the image
(2) This wavelet transform split the image into low frequency, horizontal, vertical and diagonal components
(3) Now add all the horizontal, vertical and diagonal components . Make a threshold based on the wavelet coefficients summation values and the threshold value is half of the maximum value in the summation values.
(4) Make all the coefficients to one which are greater than the threshold and suppress the remaining coefficients to zero in the summation values
(5) Now apply inverse discrete wavelet transform to the summation coefficients
(6) Now an image with only edges can be detected

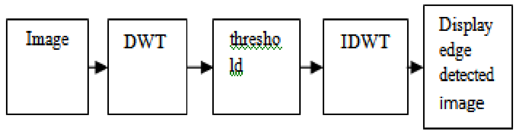


Fig.3. block diagram of wavelet based edge detection

(b) Performance metrics

(i)Visual effects*:* The quality of an image is subjective and relative, depending on the observation of the user.

(ii) Ratio of edge pixels to size of image(REPS):

If the pixel value of image is greater than the threshold value, then it is considered as edge pixel or edge point. REPS can be calculated by taking the ratio of number of edge pixels to the Size of an image[7]. This Ratio is calculated on the basis of visual perception of image. This result gives the information about the required percentage of pixels in order to represent the proper edge features in the image.

$$\mathrm{REPS}(\%) = \frac{\text{No.of Edge Pixels}}{\text{Size of an image}} \mathrm{X}\,100 \qquad (10)$$

iii) Peak Signal to Noise Ratio (PSNR): It is one of the parameters that can be used to quantify image quality.

A larger PSNR produces better image quality.

$$\mathrm{PSNR} = 20\log_{10}\frac{255}{\sqrt{\mathrm{MSE}}} \qquad (11)$$

where mean square error[12]

$$MSE = \frac{1}{mn}\sum_{y=1}^{m}\sum_{x=1}^{n}(I(x,y) - I'(x,y))^2 \qquad (12)$$

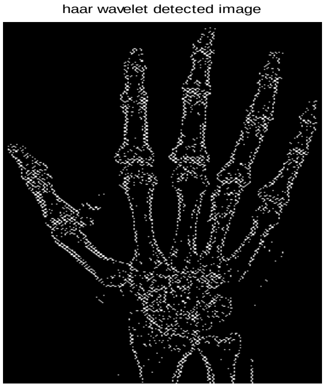
$I(x,y)$ is Original Image, $I'(x,y)$ is edge detected image

(iv)Compuatation time: It is the time taken to excute the Program

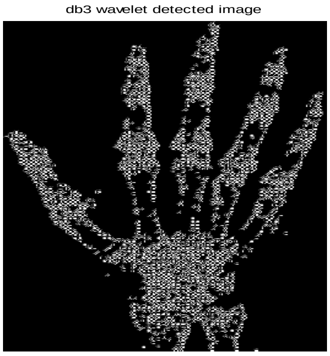
VI. RESULTS AND DISCUSSION

In this paper, right hand X-ray bone image with minute fracture at the neck of the fourth metacarpal bone and fractured foot image are considered. In order to calculate the edge features of these images, a new algorithm is implemented using MATLAB7. 9. The various orthogonal wavelet transforms are applied to detect the edges of bone in X -ray hand and foot images. “Fig. 4” and “Fig. 5” shows the images processed by these methods.

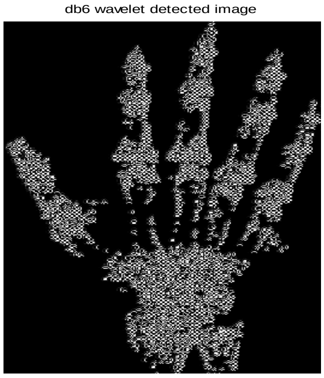
(a) original X-ray hand image



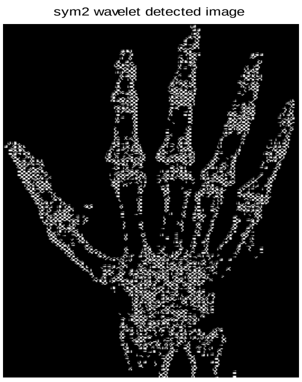
(b) haar wavelet edge detected image
edge points 16296



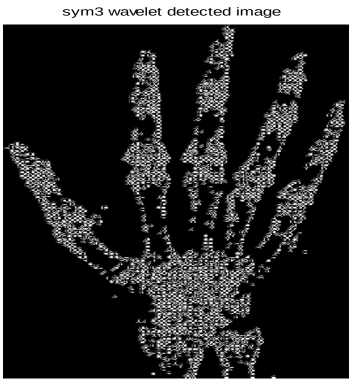
(c)db3 wavelet edge detected image
edge points 16182



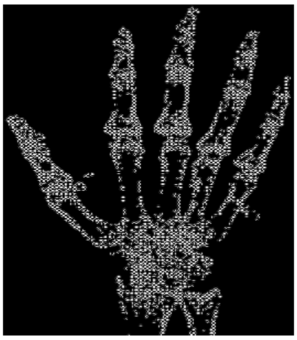
(d)db6 wavelet edge detected image
edge points 17832



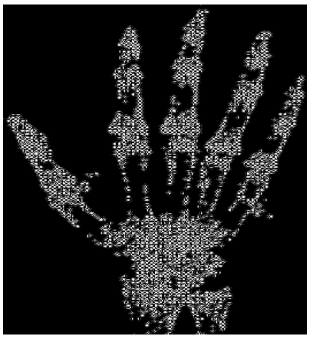
(e)sym2 wavelet edge detected image
edge points 16467



(f)sym3 wavelet edge detected image
edge points 16182



(g)coif1 wavelet edge detected image
edge points 16668



(h)coif2 wavelet edge detected image
edge points 17682

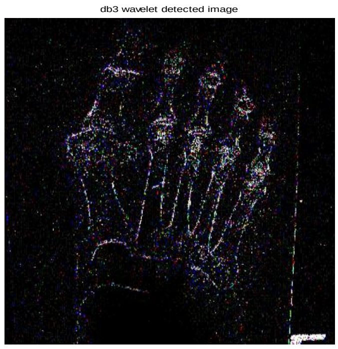
Fig.4. (a) original X- ray hand image (599x395) jpg (b)to(h) wavelet edge detected images

TABLE I: Performance metrics for edge detected X -ray hand image (599x395) jpg for various thresholds

Wavelet Type	**Threshold**	**Edge points**	**PSNR(dB)**	**Computation time(sec)**	**REPS(%)**
haar	0.03-0.06	6519	11.3733	1.4688	2.76
	0.05-0.07	12180	11.3725	1.1719	5.15
	0.09-0.1	**16296**	**11.3720**	**1.1719**	**6.89**
db3	0.03-0.06	7635	11.3731	0.9219	3.23
	0.05-0.07	10953	11.3727	1.2813	4.63
	0.09-0.1	**16182**	**11.3720**	**1.5469**	**6.84**
db6	0.03-0.06	11100	11.3727	1	4.69
	0.05-0.07	11100	11.3727	0.9219	4.69
	0.09-0.1	**17832**	**11.3718**	**1.3750**	**7.54**
sym2	0.03-0.06	7329	11.3732	1.2344	3.10
	0.05-0.07	10938	11.3727	1.2188	4.62
	0.09-0.1	**16467**	**11.3720**	**1.2344**	**6.96**
sym3	0.03-0.06	7635	11.3731	0.8750	3.23
	0.05-0.07	10953	11.3727	1.2656	4.63
	0.09-0.1	**16182**	**11.3720**	**1.5156**	**6.84**
coif1	0.03-0.06	9822	11.3729	1.2344	4.15
	0.05-0.07	9822	11.3729	1.2031	4.15
	0.09-0.1	**16668**	**11.3720**	**1.4063**	**7.04**
coif2	0.03-0.06	8370	11.3731	1.1719	3.54
	0.05-0.07	11064	11.3727	1.2813	4.68
	0.09-0.1	**17682**	**11.3719**	**1.5156**	**7.47**

(a) original X-ray foot image

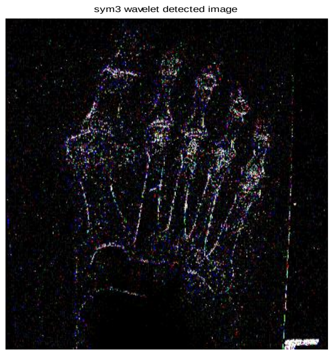
(b) haar wavelet edge detected image
edge points 218725



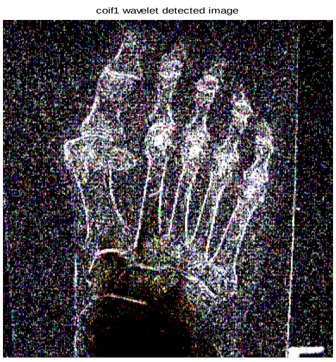
(c)db3 wavelet edge detected image
edge points 222065

(d) db6 wavelet edge detected image
edge points 234356

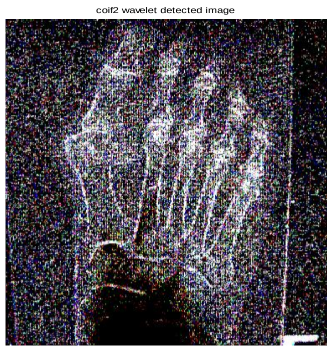
(e) sym2 wavelet edge detected image
edge points 224690



(f) sym3 wavelet edge detected image
edge points 222065



(g) coif1 wavelet edge detected image
edge points 223876



(h) coif2 wavelet edge detected image
edge points 232357

Fig.5. (a) original X -ray foot image (2048x1536) jpg (b)to(h) wavelet edge detected images

TABLE 2: Performance metrics for edge detected X- ray foot image(2048x1536) jpg for various thresholds

Wavelet Type	**Threshold**	**Edge points**	**PSNR (dB)**	**Computation time(sec)**	**REPS(%)**
haar	0.03-0.06	98751	7.3292	13.1563	3.14
	0.05-0.07	131113	7.3291	14.8281	4.17
	0.09-0.1	**218725**	**7.3288**	**11.4063**	**6.95**
db3	0.03-0.06	82305	7.3293	9.656	2.62
	0.05-0.07	126951	7.3291	13.9531	4.04
	0.09-0.1	**222065**	**7.3287**	**17.3287**	**7.06**
db6	0.03-0.06	118496	7.3291	10.0625	3.77
	0.05-0.07	143744	7.3290	15.8125	4.57
	0.09-0.1	**234356**	**7.3287**	**17.4063**	**7.45**
sym2	0.03-0.06	73570	7.3293	9.8281	2.34
	0.05-0.07	136024	7.3291	15.4688	4.32
	0.09-0.1	**224690**	**7.3287**	**18.2969**	**7.14**
sym3	0.03-0.06	82305	7.3293	9.7813	2.62
	0.05-0.07	126951	7.3291	13.9375	4.04
	0.09-0.1	**222065**	**7.3287**	**16.9375**	**7.06**
coif1	0.03-0.06	95098	7.3292	13.8281	3.02
	0.05-0.07	128215	7.3291	15.4531	4.07
	0.09-0.1	**223876**	**7.3287**	**18.5781**	**7.12**
coif2	0.03-0.06	102248	7.3292	12.75	3.25
	0.05-0.07	133861	7.3291	15.7031	4.26
	0.09-0.1	**232357**	**7.3287**	**15.9219**	**7.39**

TABLE 3: Performance metrics for edge detected X- ray hand and foot image

Image	**Wavelet type**	**Threshold**	**Edge points**	**PSNR(dB)**	**Computation time(sec)**	**REPS(%)**
hand image 599x395 jpg	haar	0.09-0.1	16296	11.3720	1.1719	6.89
	db3	0.09-0.1	16182	11.3720	1.5469	6.84
	db6	0.09-0.1	17832	11.3718	1.3750	7.54
	sym2	0.09-0.1	16467	11.3720	1.2344	6.96
	sym3	0.09-0.1	16182	11.3720	1.5156	6.84
	coif1	0.09-0.1	16668	11.3720	1.4063	7.04
	coif2	0.09-0.1	17682	11.3719	1.5156	7.47
foot image 2048x 1536 jpg	haar	0.09-0.1	218725	7.3288	11.4063	6.95
	db3	0.09-0.1	222065	7.3287	17.3287	7.06
	db6	0.09-0.1	234356	7.3287	17.4063	7.45
	sym2	0.09-0.1	224690	7.3287	18.2969	7.14
	sym3	0.09-0.1	222065	7.3287	16.9375	7.06
	coif1	0.09-0.1	223876	7.3287	18.5781	7.12
	coif2	0.09-0.1	232357	7.3287	15.9219	7.39

From the “Fig. 4” and “Fig. 5”,it is clear that the resluts of haar wavelet is bettter than other wavelets for both hand image and foot image interms of visual perception. The detected edgepoints are shown for each image (shown in “Fig 4” and “Fig. 5”). The number of edge points and REPS is less for haar wavelet when compared to other wavelets but visual perception is good for haar wavelet. Though the number of edge points and REPS is high for the other wavelets compared to haar wavelet, but their visual quality is less due to false edge points. The PSNR for haar wavelet is bit high for both hand and foot image. The computation time is less for the Haar wavelet transform when compared with other wavelets which are shown in the Table.3. In this paper,the threshold is set as 0.09-0.1 as it produces more edge points. The haar wavelet is the best due to its visual quality, high PSNR, less computation time when compared with other wavelets.

VII. CONCLUSIONS

In this paper various orthogonal wavelets are investigated for the precise analysis of X-ray image for detecting the minute fractures. Among all the wavelets, the haar wavelet is faster and best due to its good visual quality. The multiresolution analysis of wavelet transform can improve the quality of edge detection. Since the threshold is selected directly from the coefficients of wavelet transform, the quality of edge detection of X -ray bone image is very good. The PSNR value of edge detected hand image based on haar wavelet is 11.3270 dB and for foot image it is 7.3288 dB which is high compared to the other wavelets. The computation time of haar wavelet for hand image is 1.1719 sec and for foot image it is 11.4063 sec which is less when compared to the other wavelets. The haar wavelet gives best result when compared with other orthogonal wavelets for detecting the minute fractures of an X-ray image.

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